3.6 Factoring Trinomials by Decomposition.

Example 1 Multiplying Two Binomials with Positive Terms

Expand: $(3d + 4)(4d + 2)$

SOLUTION

CHECK YOUR UNDERSTANDING

Factoring a Trinomial by Decomposition

Using example:
to multiply (3h + 4)(2h + 1)

$= 3h(2h + 1) + 4(2h + 1)$

$= 6h^2 + 3h + 8h + 4$

$= 6h^2 + 11h + 4$

To factor $6h^2 + 11h + 4$, using decomposition, we reverse the steps from above.

Notice that:
- the coefficients of the h-terms have the product: 3(8) = 24
- the product of the coefficient of the h^2-term and the constant term: 6(4) =24

So, to factor $6h^2 + 11h + 4$, we decompose the h-term and write it as a sum of two terms whose coefficients have a product of 24.

Factors of 24	Sum of Factors
1,24	25
2,12	14
3,8	11
4,6	10

The two factors that have a sum of 11 are 3 and 8.

We remove a common factor from the 1st pair of terms, and from the 2nd pair of terms:

$(6h^2 + 3h) + (8h + 4) = 3h(2h + 1) + 4(2h + 1)$

common to both

So, this equals: (3h + 4)(2h + 1)

Example 4 Factoring a Trinomial by Decomposition

Factor.

a) $3s^2 - 13s - 10$

Step One: multiply the first and last numbers $3(-10) = -30$

Step Two: Find all possible factors of -30

$-15, 2$ $-3, 10$ $1, -30$ $-5, 6$

$15, -2$ $3, -10$ $-1, 30$ $5, -6$

Step Three: Which pair adds to make -13?

$-15, 2$

Step Four: Replace -13 with the two factors

$3s^2 - 15s + 2s - 10$

Step Five: Factor the first two terms and then the last two

$3s(s - 5) + 2(s - 5)$

$(s - 5)(3s + 2)$ ✓

CHECK YOUR UNDERSTANDING

SOLUTION

b) $6x^2 - 21x + 9$

$6x^2 - 18x - 3x + 9$

$6x(x-3) - 3(x-3)$

$(x-3)(6x-3)$

54
30 · 24
10 · 18

$-54, -1$

$-27, -2$

$-9, -6$

$-18, -3$

Practice Questions:

Pg. 178 #15 (Factor by decomposition)

Step One: multiply the first and last numbers

Step Two: Find all possible factors

Step Three: Which pair adds to make the middle term?

Step Four: Replace the middle term with the two factors

Step Five: Factor the first two terms and then the last two